chemical power resided. He found that the green matter did not possess it in itself, that it operated in the cells of the parenchyma, and that the vessels and pores of the cuticle have a useful influence in the phenomenon, so as to increase the quantity of oxygen gas disengaged. When solar light is received on the superior surface of leaves immersed in spring water, the quantity of oxygen gas disengaged is, in the same time and under similar circumstances, two or three times greater than when it is received on the inferior surface. The same difference may be observed in the diffused light, by means of the leaves of *Camellia japonica*, Portugal laurel, and some others which, when kept during some time in the dark in spring water, give out, when brought into the light, bubbles of oxygen gas through the central vessels of the footstalk.

The following is a brief recapitulation of the facts which the author has attempted to prove in this paper:—

1. The theories advanced to explain the curling up of tendrils, do not agree with the experiments made on those of the *Tamus communis*, this phenomenon being the result of a vital irritability acted upon by chemical agents.

2. The direction of the green parts of plants towards the light is not the result of an attraction properly so called.

3. The bending outwards of slit stems is due to the elongation of the cellular tissue by endosmose of water and the resistance of the cuticle.

4. The quantity or rapidity of endosmose is not influenced by heat or light.

5. Light is the only agent of the natural position of leaves and of their turning over when inverted. The blue are the most, the red the least active rays.

6. Light does not act in this case by a physical attraction or repulsion, properly so called.

7. The turning over of leaves takes place sometimes by a torsion of the footstalk, sometimes by a curling of the flat part.

8. The blue rays appear to be the most, and the red the least active in operating the turning over of leaves.

9. The exhalation of leaves is much increased when their inferior surface is exposed to light.

10. The decomposition of carbonic acid and the disengagement of oxygen gas are, under the same circumstances, considerably diminished.

6. "On the Solution of Linear Differential Equations." By Charles James Hargreave, Esq., B.L., F.R.S., Professor of Jurisprudence in University College, London.

1. By the aid of two simple theorems expressing the laws under which the operations of differentiation combine with operations denoted by factors, functions of the independent variable, the author arrives at a principle extensively applicable to the solution of equations, which may be stated as follows:—"if any linear equation $\varphi(x,\mathrm{D}).u=\mathrm{X}$ have for its solution $u=\psi(x,\mathrm{D}).\mathrm{X}$, this solution being

so written that the operations included under the function ψ are not performed or suppressed, then $\phi(\mathrm{D},-x).u=\mathrm{X}$ has for its solution $u=\psi(\mathrm{D},-x).\mathrm{X}$." The solution thus obtained may not be, and often is not, interpretable, at least in finite terms; but if by any transformation a meaning can be attached to this form, it will be found to represent a true result.

An important solution immediately deducible from this principle is given by Mr. Boole in the Philosophical Magazine for February 1847, and is extensively employed in the present paper. It is immediately obtained by making the conversion above proposed in the general equation of the first order and its solution.

2. By the use of this theorem and the general theorems above referred to, the solution of the equation

$$\mathrm{D}^2u+2\mathrm{Q}.\mathrm{D}u+\left(c^2+\mathrm{Q}^2+\mathrm{Q}'-\frac{m(m+1)}{x^2}\right)u=\mathrm{P},$$

is found in the form

$$u=x^m\varepsilon^{-\int\mathrm{Q}dx}(\mathrm{D}^2+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+c^2)^{-m}(x^{-(m-1)}.\varepsilon^{\int\mathrm{Q}dx}.\mathrm{P})\};$$

of which various particular cases and transformations are given and discussed; including the well-known forms

$$\mathrm{D}^2u+\frac{2m}{x}\mathrm{D}u\pm c^2.u=\mathrm{P},$$

$$\mathrm{D}^2u+b\mathrm{D}u+\left(c^2-\frac{m(m-1)}{x^2}\right)u=\mathrm{P},$$

$$\frac{d^2u}{dz^2}+\left(\frac{c}{2n-1}\right)^2z^{-\frac{4n}{2n\pm1}}.u=0,$$

and extensions of these forms.

The application of the process to equations of the third and higher orders gives rise to solutions of analogous forms; and in particular the equation

$$(a_nx+b_n)\mathrm{D}^nu+\ldots+(a_1x+b_1)\mathrm{D}u+(a_0x+b_0)u=\mathrm{X}$$

is solved in the form

$$u=(a_n\mathrm{D}_n+\ldots+a_1\mathrm{D}+a_0)^{-1}\varepsilon^{\frac{b_n}{a_n}\mathrm{D}}(\mathrm{D}-\alpha)^{\mathrm{A}}(\mathrm{D}-\beta)^{\mathrm{B}}\ldots$$
$$\left(x^{-1}\left\{\varepsilon^{-\frac{b_n}{a_n}\mathrm{D}}(\mathrm{D}-\alpha)^{-\mathrm{A}}(\mathrm{D}-\beta)^{-\mathrm{B}}\ldots\mathrm{X}\right\}\right),$$

where $\dfrac{b_nz^n+b_{n-1}z^{n-1}+\ldots}{a_nz^n+a_{n-1}z^{n-1}+\ldots}=\dfrac{b_n}{a_n}+\dfrac{\mathrm{A}}{x-\alpha}+\dfrac{\mathrm{B}}{x-\beta}+\ldots$;

and by the application of the theorems first referred to, a still more general form is solved.

The solutions above-mentioned are subject to the important restriction that m, A, B, &c. (denoting the number of times that the

operations are to be repeated) must be integer; but in the subsequent part of the paper, a mode is suggested of instantaneously converting these solutions into definite integrals not affected by the restriction.

3. The interchange of symbols above suggested frequently renders available forms of solution which otherwise would not be interpretable in finite terms. The operation $(\phi \mathrm{D})^m$ is not intelligible if m be a fraction; but if by any legitimate process this be changed into the factor $(\phi(-x))^m$, the restriction ceases to operate. By the application of this principle, solutions of a simple character are obtained for (b being integer),

$$(x^2+c^2)\mathrm{D}^2u-2ax\mathrm{D}u+b(2a-b+1)u=\mathrm{P},$$

$$\frac{d^2u}{dt^2}-\frac{b(b+1)}{\cos^2 t}u=\mathrm{P},$$

$$\frac{d^2u}{dt^2}-b(b+2)\frac{1}{(1-t^2)^2}.u=\mathrm{P},$$

$$\phi x.\mathrm{D}^2u+\psi x.\mathrm{D}u+(\psi' x-\phi'' x)u=\mathrm{P}.$$

4. The advantages of the forms above given in this particular, that the number and order of the operations in the solution are expressed *generally*, and not by a series of substitutions involving changes of the variable as in the ordinary mode of solving Riccati's equation, appear more clearly in the application to partial linear differential equations. Thus, the equation

$$\frac{d^2u}{dx^2}+\frac{2n}{x}\frac{d^2u}{dxdy}+\left(\frac{n^2}{x^2}-k^2\right)\frac{d^2u}{dy^2}-\frac{n}{x^2}\frac{du}{dy}-\frac{m(m-1)}{x^2}.u=\psi(x,y),$$

which may be solved by m successive substitutions, receives its solution in the general form

$$u=x^m\varepsilon^{-n\log x.\mathrm{D}'}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{m-1}$$
$$\left\{x^{-1}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{-m}\{x^{-(m-1)}\varepsilon^{n\log x\mathrm{D}'}\psi(x,y)\}\right\};$$

which exhibits at a glance all the successive processes to be performed upon $\psi(x,y)$ in order to arrive at the result. It will be observed that the process $\varepsilon^{\phi x.\mathrm{D}'}$ performed upon ψy denotes $\psi(y+\phi x)$. Among other results worthy of notice on this branch of the subject may be noticed the solution of

$$\frac{d^2u}{dpdq}+\frac{a}{p+q}\left(\frac{du}{dp}+\frac{du}{dq}\right)+\frac{a(a-1)-m(m-1)}{(p+q)^2}=\phi(p,q)$$

(solved by Euler in a series when there is no second term); viz.

$$u=x^{m-a}(\mathrm{D}^2-\mathrm{D}'^2)^{m-1}\left\{x^{-1}(\mathrm{D}^2-\mathrm{D}'^2)^{-m}\{x^{a-m+1}.\psi(x,y)\}\right\};$$

ψ being determined from ϕ by the equations $\frac{p}{q}=x\pm y$; and the solution of

$$(a_n x+b_n)\frac{d^n u}{dx^n}+(a_{n-1}x+b_{n-1})\frac{d^n u}{dx^{n-1}dy}+..+(a_0 x+b_0)\frac{d^n u}{dy^n}=\phi(x,y)$$

which is readily deduced from the solution of the corresponding form in ordinary equations.

5. The character of most of the solutions may be described as follows: they consist in the performance (repeated m times) of operations of the form ϕD upon the second side X; multiplication by the factor x^{-1}; and the performance (repeated $m-1$ times) of the inverse operation $(\phi D)^{-1}$; and it will be seen that, in all cases where $X=0$, it is sufficient to perform the direct operation ϕD a single time.

It is a remarkable phenomenon connected with the solutions last mentioned, that they are instantaneously convertible into definite integrals by changing ϕD into ϕz, multiplying by ε^{zx}, changing x^{-1} into D'^{-1} (D′ denoting differentiation with regard to z), and assigning proper limits for the integral. In this manner definite integrals are immediately found for

$$D^2u+2Q.Du+\left(Q^2+Q'-c^2-\frac{m(m-1)}{x^2}\right)u=0,$$

$$D^n u+\frac{u}{x}=0,$$

$$D^n u+x.u=0,$$

$$(a_n x+b_n)D^n u+..+(a_0 x+b_0)u=0,$$

and other forms.

6. The application of the principle above stated to equations of finite differences gives solutions for the equations

$$(a_n x+b_n)u_{x+n}+....+(a_1 x+b_1)u_{x+1}+(a_0 x+b_0)u_x=Q_x,$$

$$(a_n x+b_n)\Delta^n u_x+....+(a_1 x+b_1)\Delta u_x+(a_0 x+b_0)u_x=Q_x;$$

and where the number of operations to be performed is denoted by a fraction, solutions are found in the form of definite integrals.

The solution of the first when $Q_x=0$ is

$$u_x=c_1\int_0^\alpha(a_n v^n+..a_1 v+a_0)^{-1}v^{b_0}(v-\alpha)^{A_1}(v-\beta)^{A_2}...v^{x-1}dv$$
$$+c_2\int_0^\beta(a_n v^n+..a_1 v+a_0)^{-1}v^{b_0}(v-\alpha)^{A_1}(v-\beta)^{A_2}...v^{x-1}dv$$
$$+\ \&c.;$$

and that of the second is somewhat similar.

From some investigations effected by interchanging the symbols x and D in the solution of the general linear equation in finite differences of the first order, it would seem that definite summations may be used to represent the solutions of certain forms of equations. Thus a partial solution of

$$\varepsilon^{-x}.u-\mathrm{D}^n u=c$$

is $c\Sigma(\Gamma z)^n\varepsilon^{zx}$ from $z=-\alpha$ to $z=0$.

7. In attempting the solution of some equations by means of successive operations, not consisting exclusively of D combined with constants, but involving also functions of x, the only result which appeared to the author worthy of notice is the solution of

$$\mathrm{D}^2u+b\mathrm{D}u+c^2u-n(n+1)\frac{u}{\cos^2 x}=\mathrm{X};$$

from a particular case of which, the general solution of Laplace's equation,

$$\frac{d}{d\mu}\left((1-\mu^2)\frac{du}{d\mu}\right)+\frac{1}{1-\mu^2}\frac{d^2u}{dy^2}+n(n+1).u=0,$$

may be found in the simple form

$$u=\varepsilon^{\tan^{-1}(\mu\sqrt{-1})\frac{d}{dy}}\left(\frac{d}{d\mu}\right)^n\left\{(1-\mu^2)^n\phi\left(y-2\tan^{-1}\mu\sqrt{-1}\right)\right\},$$

with a similar function using $-\sqrt{-1}$ for $\sqrt{-1}$.

7. "Researches on the Function of the Intercostal Muscles and on the Respiratory Movements, with some remarks on Muscular Power, in Man." By John Hutchinson, M.R.C.S. Communicated by Sir Benjamin Brodie, Bart., F.R.S., &c.

The object of this paper is to demonstrate by models and dissections the action of the intercostal muscles.

After premising an account of the views of several eminent physiologists, and in particular those promulgated by Haller, the author shows that they resolve themselves into the general opinion that the scalene or other muscles of the neck fix the first rib, in order to enable the two sets of intercostal muscles to act either separately or conjointly, as inspiratory or expiratory muscles. He then proceeds to state the proofs that the intercostal muscles possess an action which is independent of any other muscle, and also independent of each other, so that any of the twelve ribs may be elevated or depressed by them either separately or conjointly. He demonstrates the nature of this action by means of models, producing oblique tensions between levers representing the ribs, and allowing of rotation on their centres of motion; and he shows that such tension in the direction of the external intercostal muscles, elevates both the levers until the tension ceases, or the position of the bars by proxi-